PROOFS ASSIGNMENT

ENGLISH 125-64
RAFE KINSEY

Introduction

In this assignment, you will practice writing a mathematical proof. Proof is one of the fundamental concepts in pure mathematics—it separates mathematics from almost all other fields, in that a proof shows with 100% absolute certainty that something is true.[1] This assignment serves as the culmination of the first part of the course, where we've studied *mathematical logic*, understanding the building blocks of how mathematical statements are put together.

As we move forward in the course, what we've learned from this section will help us in at least two ways:

- The logic we use in this formal setting will help us understand the role of logic in making arguments in a variety of less formal contexts.
- The abstract methods we've used here, especially the logical notation we've used, will help us be precise as we move from mathematics to the messier world of natural language, as well as to the topics you will choose for your final project.

More generally, the *mathematical maturity* and *sophistication* you've developed over the past few weeks while working on—and sometimes struggling with—Devlin's book will pay off in the remainder of this course and throughout college, as you learn to deal with difficult concepts and ideas. (And don't worry if you still feel confused about some of the harder aspects of logic. If this were a math course, we'd spend more time ensuring everyone understands everything, but this is a writing course, so the focus is on the process, not the specific concepts learned.)

I'm giving you this assignment for a few reasons. First, I hope you will feel some satisfaction and accomplishment with being able to write a full mathematical proof after only two weeks of studying logic! Second, this will give you an opportunity to make a precise and rigorous logical argument with no holes in it—so that as you move towards less mathematical writing, you'll keep in mind the importance of logic. Finally, this assignment will require you to *explain* clearly what you're doing, an important task since clear explanation is one of the foundations of good writing.

The Proof

Take one of the following mathematical proofs[2]:

(a) Prove that $\sqrt{7}$ is an irrational number. (This is a variation on the problems we've done class.)

Date: Monday, September 23, 2013.

[1]Compare this with a scientific experiment. In science, an experiment can show that a theory is extremely likely to be true. But it's always possible that there was a statistical fluke. Of course, there's nothing at all wrong with scientific proof; it forms the basis for a huge amount of human knowledge. But it's useful to keep in mind the distinction between mathematical proof—also called *deductive* proof—and scientific proof, called *inductive proof.* (For those of you who have heard of "mathematical induction"—if not, don't worry—that is a different sense of the word *induction* than the sense I'm using here.)

[2]Remember, this is a writing course, not a math course, so the focus is on how well you explain concepts, not how hard of a math problem you choose. But if you're interested in pure math, challenging yourself with a harder problem might be interesting!

(b) Prove that the "mutilated chessboard" has no domino covering. (We did this the first day.)
(c) Prove Devlin 3.4.1 #9: If the Goldbach Conjecture is true, then every odd natural number greater than 5 is the sum of three primes numbers.
(d) (Harder): If you are interested in learning about different types of infinities, you could do Cantor's diagonalization argument that there are more irrational numbers than rational numbers. If you're interested in doing this, email me and I'll send you references.
(e) (Harder) If there's a short proof not on this list that you want to do—for example, a proof from a math class you're taking—that's fine, so long as you check with me first.

You'll choose one of these proofs and email me by *Wednesday*'s class—or ideally sooner—letting me know which you have chosen.

First try to understand the proof. That's often *hard*. You'll have to work through it carefully, read it several times. It's *fine* to ask classmates or others for help understanding the proof (or to look for help online or in a book)—although you should try first to make sense of it yourself. I've chosen some proofs that we've done in class already, so hopefully you already understand these.

Then think about how you would present the proof, especially how you would present the proof to the following different *audiences*. How much explanation would you give? Would you skip any steps? Would you give examples? What would you have to define?

(1) Suppose there was a supercomputer that could read and verify the validity of proofs, and you were submitting this to the supercomputer.
(2) Suppose all you had to do was convince an experienced mathematician that the proof was correct.
(3) Suppose you were a mathematician, and you'd just discovered the proof. You know that your memory isn't great, so you decide to write down the proof in case you want to see it again 10 years from now.
(4) Suppose you were a student in a math course, and you want to write down the proof in a way that shows the professor or grader that you understand the proof. (Note: this is similar to what you've done for the HW assignments so far.)
(5) Suppose you're a professor writing a textbook for students in a math course, and you want to write the proof in a clear way that students with a background in the course can understand. (Note: this is what Devlin is doing.)
(6) Suppose you're writing an article or book about math to the general public, and you want to write the proof in a way that explains concepts the general public might not be familiar or comfortable with. (Note: this is similar to what Hardy has done in his *Mathematician's Apology*. Devlin, to some extent, also falls into this category.)
(7) (Can you imagine any other scenarios? Can some of these scenarios be further split into different cases?)

Once you feel like you really understand the proof, try to work out the proof on paper *without* looking at any notes. If you need to, if there's a step you're missing, it's okay to look back at your notes, or to consult a reference you might have found online. Then try to do it *again* without your notes.

Once you've done this, try writing up the proof as if you were a student in a math course (4). You should do the writing up of the assignment *alone*, and you *shouldn't* look at any of the other references you have. The point is that if you understand the proof, you should be able to write it down by yourself. Feel free to handwrite this part; it's just like a homework assignment. (You'll hand this in at the end.)

Optional: If you hand this in by class on Wednesday, September 25 or in the mailbox of my office door in 5832 East Hall by noon on Thursday, September 26, I'll take a look at it and let you know by email if there are any mathematical mistakes by Friday, September 27, at 1pm. (Make a copy to keep for yourself, since you won't get the physical copy back until the polished draft is due.)

The Assignment

Then, for the major part of the assignment, write the same proof up as if you were writing to the general public (i.e., audience 6 in the list above). Your exposition should be *self-contained*: that means that you should make sure not only to include the proof, but also state and explain the problem and define any relevant terms or concepts. As you present the proof, you might want to explain not only what you're doing but *how* and/or *why* you're doing it. (Remember, you're writing for a general audience, so they will need more explanation than an audience of, say, mathematicians.) It might also make sense to provide a bit of relevant *context*: what is the significance of the problem?[3]

Once you start this part of the assignment, feel free to look at *your* writeup of the proof, but don't consult any other written version of the proof.[4] The reason I'm forbidding you from looking at other written references is so that you can avoid plagiarism: the point of the assignment is for *you* to work out how to explain the proof effectively, not to copy someone else's words. (Remember, if you have any questions about plagiarism, in this or any other class, *ask your instructor!*)

It's fine for you to look at examples of expositions of *other* proofs; indeed, I'd encourage you to do this. Look at how they explain concepts, define relevant terms, offer examples, etc.[5] You should feel free to show this writeup to friends or family members not in the class, to see if your explanation makes sense to them. (You could also try explaining the proof to them out loud before writing, to see if your explanations make sense. Or do both!) We'll also do a peer review in class.

Requirements

Formatting and Length. Your assignment should be typed, in 12-point font and *single-spaced*.[6] As you write equations, you'll want to think about effectively using *white-space*. Notice how Devlin and Hardy include some mathematics in the text of a paragraph but also have some equations on separate lines. This is often quite effective and helps make your writing more readable. You might also want to include diagrams, depending on the proof you choose. (Certainly, you'd want to include diagrams for the mutilated chessboard problem.) Depending on which proof you choose, your writeup should be about 1-3 pages (single-spaced).

Acknowledgment of Sources and Collaboration. At the end of your assignment, write a paragraph letting me know who you collaborated with on the assignment and what resources you consulted. E.g., you might write something like: *To understand the proof, I took a look at Devlin's book and talked about it with Bob. I wrote the homework version of the proof on my own, and consulted Rafe to see if I understood it correctly. I wrote the exposition version of the proof by myself, and then it was peer-reviewed by Sarah and Josh. While I was working on the exposition,*

[3]This might differ based on the proof you choose. Is what you're proving mathematically significant? Is there a historical significance? Is it significant because the proof shows some useful technique?

[4]The only exception to this is that you can look at other references to understand the context of the problem, as discussed in the previous footnote. If so, don't look at the proof itself.

[5]In addition to Devlin's book and Hardy's book, I'd encourage you to go to the Shapiro Science library and look at some math textbooks. How do they explain things in their proofs? You might also look at some expositions of mathematical proofs online.

[6]If you're planning on taking a number of courses in math or computer science, you might want to learn to use the program LaTeX, which provides elegant typesetting of math. Google it or feel free to ask for more. Otherwise, you can either use the equation editor in MS Word *or* leave blank spaces in your word document and *neatly* hand-write just the symbols and/or any diagrams you use.

I took a look at Hardy's book to see how he explained mathematical concepts. Finally, I had my roommate look over the final version of my draft before I submitted it.

Reflection. As we continue to read math and other technical material in the course, think about the challenges you experienced in writing this proof. At the bottom of your final version, write a paragraph reflecting on the assignment. What was easy about it? What was hard?

Peer Revision. A *polished draft* of your assignment is due in class on Monday, September 30. In class, we'll do *peer review*, a really important part of the course—both for helping you improve your papers but *also* in helping you develop good editing skills. Make sure to bring *three copies* of the assignment to class.

Final Assignment. After the peer review, you'll edit your assignment, and then submit the final version to me. This will be due *Tuesday, October 8* at 4pm, in the mailbox on the door of my office, 5832 East Hall. You should submit the following items in hard copy, *stapled together*:

- Your final exposition (typed)
- The homework version of the proof (handwritten is fine)
- The polished draft you submitted for peer revision.

Also, please email me a copy of your final exposition *as a pdf* by Tuesday, October 8, 4pm, with the subject "English 125: Proofs Assignment". (If you have hand-drawn figures or equations, it's alright if those aren't in the emailed version.)

Grading. This is your first major writing assignment for the course, and will constitute 5% of your grade. In the next few weeks, we'll be talking more about the criteria of good writing in this class. For now, focus on clear and concise explanation and natural-sounding writing.

Timeline

- Wednesday, 9/25 2:30pm. Email me letting me know which proof you're doing.
- (Optional) Thursday, 9/26 noon. Hand in a copy of your writeup to me; I'll let you know if there are any mathematical mistakes by Friday.
- Monday, 9/30 in class. Bring *three* copies of a *polished* draft to class for peer revision.
- Wednesday, 10/2 2:30. Email your peer revision comments to your peer partners; cc me on your email.
- Tuesday, 10/8 by 4pm. Hand in your final version to me, in the mailbox on the door of my office, 5832 East Hall, and email me a copy as a pdf.

Please make sure to follow any directions. If you have questions, let me know!